0.1 Proof of Equivalence of Analyses

It is sufficient to show that for each configuration and each statement, all transfer functions of $\mathcal{A}1$, $\mathcal{A}2$, $\mathcal{A}3$, and $\mathcal{A}4$ compute the exact same information (only represented in different ways by the four analyses). Let valid configuration $c \in [\![\psi]\!]$ and "ϕ: `S`" (i.e., statement S with feature constraint ϕ) be given. Now, we have two cases, depending on whether or not $c \in [\![\phi]\!]$.

- <u>Case $c \in [\![\phi]\!]$:</u>
  - $\mathcal{A}1$: Since $c \in [\![\phi]\!]$, we get that: $\ell' = f_S(\ell)$.
  - $\mathcal{A}2$: Here, we have that $\ell' = \begin{cases} \ell & c \in [\![\neg\phi]\!] \\ f_S(\ell) & c \in [\![\phi]\!] \end{cases}$ which means that $\ell' = f_S(\ell)$, as required.
  - $\mathcal{A}3$: In this case, the output of the transfer function is $x' = (c \mapsto \begin{cases} \ell & c \in [\![\neg\phi]\!] \\ f_S(\ell) & c \in [\![\phi]\!] \end{cases}, ...)$ which means that the information computed for configuration c is $x'(c) = f_S(\ell)$, as required.
  - $\mathcal{A}4$: Here, the output of the transfer function is $x' = ([\![\psi \wedge \phi]\!] \mapsto f_S(\ell), [\![\psi \wedge \neg\phi]\!] \mapsto \ell, ...)$. However, since $c \in [\![\psi]\!]$ and $c \in [\![\phi]\!]$, we conclude that $c \in [\![\psi \wedge \phi]\!]$ and hence that the information computed is $x'[c] = f_S(\ell)$, as required.
- <u>Case $c \notin [\![\phi]\!]$:</u>
  - $\mathcal{A}1$: Since $c \notin [\![\phi]\!]$, we get that: $\ell' = \ell$.
  - $\mathcal{A}2$: Here, we have that $\ell' = \begin{cases} \ell & c \in [\![\neg\phi]\!] \\ f_S(\ell) & c \in [\![\phi]\!] \end{cases}$ which means that $\ell' = \ell$, as required.
  - $\mathcal{A}3$: In this case, the output of the transfer function is $x' = (c \mapsto \begin{cases} \ell & c \in [\![\neg\phi]\!] \\ f_S(\ell) & c \in [\![\phi]\!] \end{cases}, ...)$ which means that the information computed for configuration c is $x'(c) = \ell$, as required.
  - $\mathcal{A}4$: Here, the output of the transfer function is $x' = ([\![\psi \wedge \phi]\!] \mapsto f_S(\ell), [\![\psi \wedge \neg\phi]\!] \mapsto \ell, ...)$. However, since $c \notin [\![\phi]\!]$, we conclude that $c \in [\![\neg\phi]\!]$ and then that $c \in [\![\psi \wedge \neg\phi]\!]$ and hence that the information computed is $x'[c] = \ell$, as required.

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